CMU CS Academy
CPCS Math Pre-Test

Notes:

- **Only use pen or pencil and paper.**
  No calculators or computers or phones or any other electronic devices. No internet. No books or notes.

- **Circle your answers.**
  All answers must be clearly circled.

- **Show all your work.**
  All answers must show all your supporting work (and not just the final answers) to receive credit. Also: no guess-and-check.

Some formulas you may use:

Distance Formula	The distance between two points (x_1,y_1) and (x_2,y_2) is: $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
Midpoint Formula	The midpoint of the line segment from (x_1,y_1) to (x_2,y_2) is: $\left(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}\right)$
Quadratic Formula	If $ax^2 + bx + c = 0$, then: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

Some math facts you may use:

- * means "times", so 2*3 is "2 times 3" which is 6.
- ** means "power", so 2**3 is "2 raised to the 3rd power", which is 2*2*2, which is 8.
- If an expression contains different operators, evaluate them in this order (highest first):
  - ** (power)
  - *, /
  - +, -

Part 1: Using Arithmetic

1. What is the value of each expression?

    1. 10 * 2 + 3

    2. 10 + 2 * 3

    3. 10 + 2 ** 3   (recall that ** is the "power" operator)

    4. 10 ** 2 * 3   (recall that ** occurs before * in the order of operations)

    5. 10 * 2 ** 3

    6. 4 * 3 ** 2 + 2 ** 3 * 3

Part 2: Using Functions

This section uses these functions:

$f(x) = 2x + 5$ $g(y) = 3y + 2$ $h(u, v) = u^2 + v$

2. Using the given functions, what is the value of each expression?

    1. f(10)

    2. f(5) + g(3)

    3. f(3) * g(2)

    4. f(g(2))

    5. f(g(1)) + g(f(2))

    6. h(10, f(4))

    7. h(18 - f(5), h(2, 3))

Part 3: Using Formulas

3. What is the distance from (6, 7) to (0, 15)? Hint: use the provided Distance Formula.

4. What is the midpoint of the line segment from (6, 7) to (0, 15)? Hint: use the provided Midpoint Formula.

Part 4: Solving Equations

5. Solve for x: $7x - 3 = 4x + 12$

6. Solve for y: $2y^2 + 3y = 12 + 5y$ Hint: use the Quadratic Formula.

7. Solve for both x and y: $2x + 3y = 2$ and $3x + y = 10$

Part 5: Using Algorithms

8. According to [wikipedia](), "a happy number is a number which eventually reaches 1 when replaced by the sum of the square of each digit". For example, say we start with 13. We replace 13 with $1^2 + 3^2$, which is 1 + 9, which is 10. Then we repeat the process, and so we replace 10 with $1^2 + 0^2$, which is 1 + 0, which is 1. Since we reached 1, that means that 13 is a happy number.

   Not all numbers are happy. Instead of reaching 1, an unhappy number eventually reaches 4. For example, say we start with 11. We replace 11 with $1^2 + 1^2$, which is 1 + 1, which is 2. Then we repeat the process, so we replace 2 with 2^2, which is 4. Since we reached 4, that means that 11 is an unhappy number.

   The examples above both required two steps to reach 1 or 4, but sometimes we need more steps than that to reach 1 or 4.

   Using this algorithm, determine if each of the following numbers are happy or unhappy, and be sure to show all your work! Note that none of these numbers requires more than 3 steps to reach 1 or 4.

   1. 10

   2. 20

   3. 32

   4. 42

9. The gcd, or the greatest common divisor of two numbers, is the largest number that divides both numbers without a remainder. For example, gcd(30, 20) is 10, because 10 evenly divides both 30 and 20, and no number larger than 10 evenly divides both 30 and 20.

   Here is a way to find the gcd of two numbers, discovered by Euclid (a great mathematician who lived about 2500 years ago):

   To find gcd(x, y), replace x with y, and replace y with the remainder when we divide x by y. Keep doing this until y is 0. At that point, x is the answer.

   For example, to find gcd(30, 20) we replace 30 with 20, and we replace 20 with the remainder when we divide 30 by 20, which is 10. Thus, we now have gcd(20, 10), and we repeat the process. So now we replace 20 by 10, and we replace 10 by the remainder when we divide 20 by 10, which is 0. So now we have gcd(10, 0). Since y is 0, x is the answer. So the answer is 10. That means that gcd(30, 20) is 10.

   Here is how we can write that more succinctly:
   gcd(30, 20) = gcd(20, 10) = gcd(10, 0) = 10

   Using this algorithm, find each of the following gcd's, and write your answer in the same form as our "succinct" version above, showing all the intermediate steps that were required.

   1. gcd(24, 20)

   2. gcd(11, 10)

   3. gcd(33, 18)

Note: Part 6 (Problem Solving) is on the next page.

Part 6: Problem Solving

10. A right triangle with an area of 24 has a side of length 6, and its hypotenuse is of length 10. What is the length of the other side? Be sure to show your work.

11. Here is an algorithm for encoding a word:
    1. Replace each letter with the position of that letter in the alphabet (so, replace 'A' with 1, 'B' with 2, and so on).
    2. Then, add 1 to the first number, 2 to the second, and so on.
    3. Finally, replace each number except the first number with the difference of that number and the previous number.
    4. Write the numbers down separated by slashes (/).

    An example will help. Let's start with the word "CAB". Here is each step:
    1. We replace 'C' with 3, 'A' with 1, and 'B' with 2, so we now have: 3, 1, 2
    2. We now have 3+1, 1+2, 2+3, that is: 4, 3, 5
    3. Now, we replace 3 with (3-4), or -1. And then we replace 5 with (5-3), or 2. So now we have: 4, -1, 2.
    4. Finally, we add the slashes, so our final encoding is: 4/-1/2

    With this in mind, you are given the *encoded* word 3/4/-3/4 and your job is to find the original unencoded word that produced that encoding. Be sure to show your work.

End of Pre-Test.